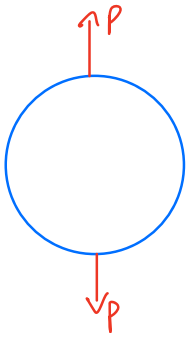
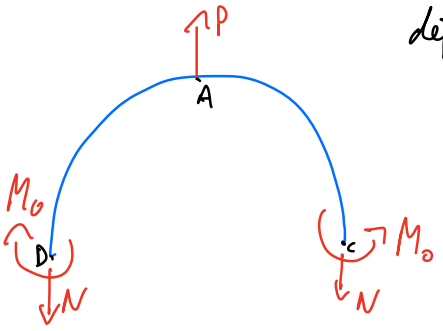
DYNAMOMÈTRE

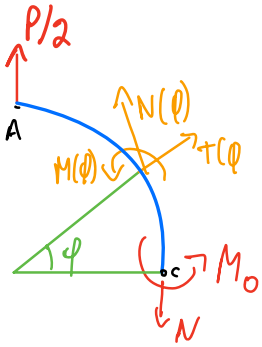


Comme au problème 11.3, hyperstatique d'ordre 3, mais symétrie réduit à $k=1$ donc 1 seule force hyperstatique

1. Manabrea pour trouver la réaction hyperstatique, → σ_{max}, par M_0 $\quad \partial U/\partial M_0 = 0$
2. Castigliano pour trouver le déplacement $\delta_A = \partial U/\partial P$



$$M_0 : \quad \frac{\partial U}{\partial M_0} = 0 \quad \text{Manabrea}$$



$$\sum M_{C_j} = 0 \quad M(\varphi) + M_0 - \frac{P}{2}(R - R\cos\varphi) = 0$$

$$M(\varphi) = -M_0 + \frac{P}{2}(R - R\cos\varphi)$$

$$\frac{\partial M(\varphi)}{\partial M_0} = -1$$

$$\frac{\partial U}{\partial M_0} = 0 = 4\,\frac{1}{EI}\int_0^{\pi/2} M(\varphi)\,\frac{\partial M(\varphi)}{\partial M_0}\,R\,d\varphi$$

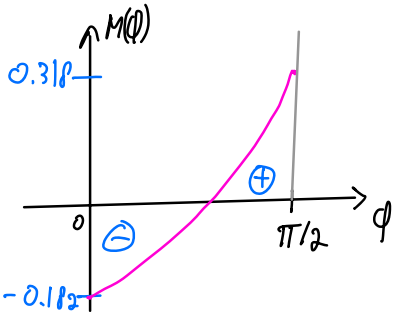
$$0 = \frac{4R}{EI}\int_0^{\pi/2} M_0 - \frac{P}{2}R(1-\cos\varphi)\,d\varphi$$

$$\boxed{M_0 = PR\,\frac{(\pi - 2)}{2\pi}}$$

CONTRAINTE MAX:

$$M(\phi) = PR\left[\frac{2-\pi}{2\pi} + \frac{1}{2}(1-\cos\phi)\right]$$

$$= \frac{PR}{2}\left(\frac{2}{\pi} - \cos\phi\right)$$



$|M(\phi)|_{MAX}$ à $\phi = \pi/2$: points A et B

$$\sigma_{max} = \frac{M_{MAX}}{W}$$

($M_{MAX} \to M(\phi = \frac{\pi}{2})$; $W \to \frac{I}{(h/2)}$)

$$= \frac{PR}{\pi}\,\frac{1}{\frac{b h^3}{12}\,\frac{2}{h}} = \frac{3}{4}\sigma_e$$

$$P = \frac{3\pi}{4}\sigma_e\,\frac{b h^2}{6}\,\frac{1}{R} = \frac{\pi\,\sigma_e\,b\,h^2}{8R} = 10'600N$$

DÉPLACEMENT DE A et B

$$\delta_A = 4.\frac{1}{EI}\int_0^{\pi/2} M(\phi)\,\frac{\partial M(\phi)}{\partial P}\,R\,d\phi$$

$$= \frac{4}{EI}\int_0^{\pi/2} \frac{PR}{2}\left(\cos\phi - \frac{2}{\pi}\right)\frac{R}{2}\left(\cos\phi - \frac{2}{\pi}\right)R\,d\phi$$

$$= \frac{PR^3}{EI}\int_0^{\pi/2} \cos^2\phi - \frac{4}{\pi}\cos\phi + \frac{4}{\pi^2}\,d\phi$$

$\hookrightarrow \frac{1}{2}(\cos 2\varphi + 1)$

$$\delta_A = \frac{PR^3}{EI}\left(\frac{1}{2}\cdot\frac{\pi}{2} - \frac{4}{\pi} + \frac{4}{\pi^2}\cdot\frac{\pi}{2}\right)$$

$$\boxed{\delta_A = \frac{PR^3}{EI}\cdot\frac{(\pi^2 - 8)}{4\pi} = 1.34\ mm.}$$

$10\ kN \quad 1.34\ mm \qquad k_{ressort} = \frac{10\,600}{1.34 \cdot 10^{-3}} = 7.9\ MN/m$